

## Production scheduling optimization for power-intensive processes with time-sensitive electricity prices

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**Abstract.** Continuous power-intensive processes in air separation plant can take advantage of optimal production planning to reduce the consumption of electricity. In this work a solution approach is developed based on a discrete-time scheduling formulation that allows modeling and optimizing operating decisions either in a fixed or a rolling horizon scheme. The main goal of this contribution is to find an optimal hourly schedule for next week that minimizes total energy consumption cost while satisfying all operational constraints. The MILP model is tested on real-world electricity price and demand input data. The results show optimal solutions for the proposed methodology with a modest computational effort considering a one-hour time grid and one-week time horizon.

**Keywords:** continuous power-intensive processes, air separation plant, scheduling, MILP model, energy consumption cost

### 1 Introduction

The issue of time-sensitive electricity costs and its impact on industrial competitiveness have become one of the most important factors for production decisions. Mainly, decision-making are required to simultaneously deal with alternative production modes and rates to lower power consumption [1], [2]. Consequently, the development of optimal production scheduling strategies has emerged as a promising alternative to reduce the consumption of electricity [3], [4].

In this study we consider air separation plant processes where the cost of energy changes hourly. The plant is assumed to participate in multiple energy markets for production: Day-ahead markets and Spot/Imbalance markets. In the day-ahead market, blocks of energy are nominated at an hourly level for the next day and are bought on a daily auction. The prices of energy are known after the auction closes, sometime around noon on the day before delivery. For a weekly time horizon, e.g. Monday to Sunday, the price and amount of power at every hour is known for Monday, whereas from Tuesday to Sunday neither the price nor the amount of power is known. However, forecasts are available for the day-ahead market for the next nine days. On the other hand, power is non-contracted in the imbalance market. It is the result of the imbalances in the physical power grid, and the attempt of the operators to match supply and demand. Prices of energy are known 15 minutes after the power is consumed.

The challenge is to predict how long the price will remain profitable so the plant has time to react or even to forecast the spikes and valleys [4], [5].

Therefore, we propose an efficient predictive and reactive solution strategy for real-world industrial scale problems to optimize participation in electricity markets under uncertainty in the operation of power-intensive air separation processes. The objective is to compare both approaches defined, predictive model and the rolling model, in terms of computational efficiency and potential economical benefits. Accordingly, a deterministic MILP model is proposed to optimal production planning of continuous power-intensive air-separation processes to efficiently adjust production operation according to time-dependent electricity pricing.

## 2 Problem statement

The scheduling problem includes minimum and maximum production rates based on the plant state, storage capacity of the plant and minimum final tank level constraint, considering that minimum final tank levels must be fulfilled depending on the day of the week of last time period of the scheduling horizon. At the same time, detailed power consumption is taken into account for the different operating modes, which follows linear correlation. Expected daily demand and hourly electricity prices are used to generate and assess different scenarios.

An important aspect in this problem is to consider that there is an operational constraint on the minimum amount of time the plant should be running in the same operation mode. When deviating from the plant, this action will affect several time periods. The plant has transition states to set-up and shut-down of equipment (ramp-up and ramp-down times) with minimum duration of 1 hour, and others states with minimum duration of 3 hours: uptime, standby time and downtime. The figure Fig.1 shows a state graph of the air-separation plant. Therefore, we proposed an explicit modeling formulation of feasible plant operational transitions and a systematic way of representing transition states.

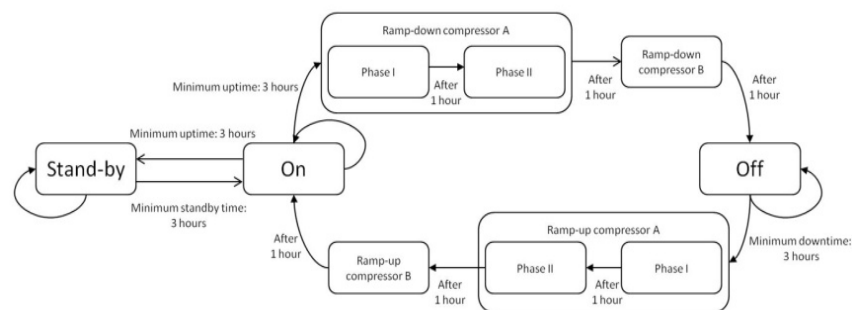


Fig. 1. A state graph of the plant

### 3 Model formulation

The MILP model developed for this work contents several components to deal with features of the problem mentioned in previous section. In section 3.1 we proposed a new network to describe the scheduling problem. In following, time representation proposed is in section 3.2, the nomenclature of the formulation is detailed in section 3.3 and operational constraints are described in section 3.4.

#### 3.1 Process State Transition Network

In this section we present a novel Process State Transition Network (PSTN) concept developed to represent specific problem features, as shown in Figure 2. States with minimum duration of 3 hours are decomposed in 3 sub-states of 1 hour each and are called initial sequential transition states, intermediate transition states, and critical transition states, respectively. Note that this decomposition occurs in stand-by, on and off operating states in which the plant can remain between 3 and undetermined amount hours.

Furthermore, sequences of transitions between different states must be met, for example to switch from off to on state the following sequence of states have to occur: RUCAP1, RUCAP2, and RUCB, with fixed duration of 1 hour in each state. From now on, each operation mode represented in scheme of Figure 2 is named states.

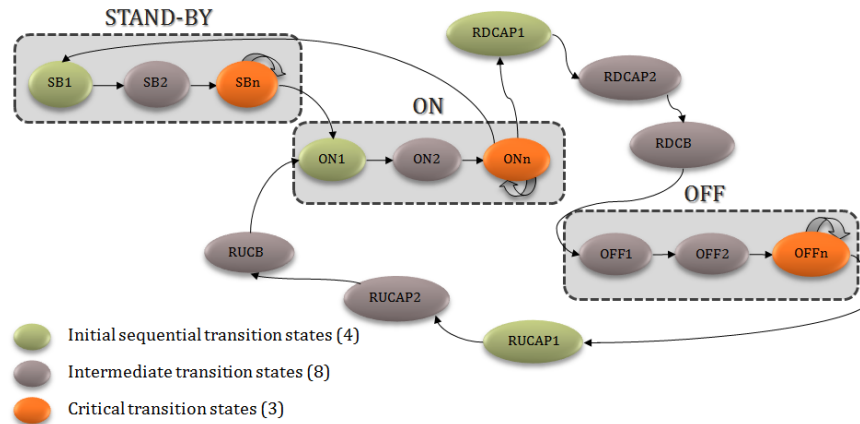


Fig. 2. Process State Transition Network

#### 3.2 Time representation

Type of procedures and the time representation used in this model are described below. Note that the scheduling model type is deterministic, in which variable the input data are energy prices and demand forecasts.

Although many continuous time models have been effectively used for this type of plant scheduling [4], [6], for operational restrictions and time characteristics required in this problem, a discrete time representation is successful. This means that the scheduling horizon is divided into fixed intervals of equal length, such that each period represents an hour on the horizon.

In that work two type of model are developed: rolling horizon and predictive model. The main difference of both approaches is the way the model is being resolved and, data and variables are being updated. Both procedures are to address the operations planning in air separation processes and are based on the forecasts previously mentioned and the constraints which are described below.

Whereas that a production plan for the week is obtained when the predictive model is solved, in the rolling horizon procedure a schedule is obtained with information updated daily. In other words, after the solution model is obtained, quantity of production, inventory levels and state transitions variables corresponding to the first day (24 hours) are fixed. Subsequently the forecasts (energy prices and demand) are updated for the following days of the week and the model is solved again. This process is repeated until complete week (168 hours).

### 3.3 Nomenclature

#### ***Sets***

|                    |  |
|--------------------|--|
| $T$ (index $t$ )   | Time periods   |
| $S$ (index $s$ )   | States   |
| $D$ (index $d$ )   | Days of a week   |
| $S^{initial}$      | Initial sequential states of On, Off and Stand-by modes      |
| $S^{inter}$        | Intermediate transition states of On, Off and Stand-by modes |
| $S^{critical}$     | Critical transition states of On, Off and Stand-by modes     |
| $S^{down-initial}$ | Initial state to ramp-down                                   |
| $S^{sup-initial}$  | Initial state to start-up                                    |
| $S^{down-inter}$   | Intermediate states to ramp-down                             |
| $S^{sup-inter}$    | Intermediate states to start-up                              |
| $LIC$              | Last intermediate states before critical states              |
| $NTS$              | Next to transition states                                    |

#### ***Parameters***

|               |   |
|---------------|---|
| $MinP_s$      | Minimum production per hour in each state       |
| $MaxP_s$      | Maximum production per hour in each state       |
| $MDTL_d$      | Minimum final tank levels at the end of the day |
| $ED_t$        | Hourly expected demand                          |
| $FPC_s$       | Fixed power consumption                         |
| $VPC_s$       | Variable power consumption                      |
| $EP_{(t,d)}$  | Energy prices forecast for a week               |
| $Qmin$        | Minimum Tank Level                              |
| $Qmax$        | Maximum Tank Level                              |
| $EP\_FIXED_t$ | Average energy price of a week                  |

|                |   |
|----------------|---|
| $I_0$          | Initial tank level                                    |
| $T\_D_{(t,d)}$ | End time of each day                                  |
| $d1_d$         | Starting day of scheduling                            |
| $nd$           | Amount of intermediate states in the shutdown process |
| $ns$           | Amount of intermediate states in the startup process  |

#### **Continuous Variables**

|           |   |
|-----------|---|
| $P_{s,t}$ | Production at time $t$ for state $s$              |
| $PW_t$    | Power consumption at time $t$                     |
| $I_t$     | Inventory available at the end of time period $t$ |
| $Cost$    | Objective function (total energy cost)            |

#### **Binary Variables**

|           |  |
|-----------|--|
| $W_{s,t}$ | Indicates whether plant operates in state $s$ during time period $t$ |
|-----------|--|

### **3.4 Constraints**

The PSTN model assigns states  $s$  to time periods  $t$  using proper binary variable  $W_{s,t}$  denoting that process is operating at a given state at every time and ensuring that all operating constraints are satisfied. Note that all the following constraints are applied in both models (predictive and rolling horizon). The difference lies in the way to solve these models, as described in the previous section.

The model minimizes the energy total cost while satisfies the start-up and shut-down restrictions, and also the constraints that concern the power consumption according to time-dependent electricity pricing schemes. In the following, we will present these constraints.

**Plant State.** The plant has to operate in a single configuration each hour, so Eq. (1) forces the plant to be in a single production mode each period.

$$\sum_s W_{s,t} = 1 \quad \forall t \in T \quad (1)$$

**Sequential Transition States.** Eq. (2)-(4) force to  $W_{s,t}$  variable that indicates the occurrence of a configuration change to start-up or shut-down equipment. If operating point of the plant in the time period  $t$  is the initial state of an operating state (on, off and stand-by), then at time  $t + 1$  and  $t + 2$  have to operate in the corresponding states, intermediate and critical, respectively (see Eq.(2)).

$$W_{s,t} = \sum_{s' \in S^{inter}} W_{s',t+1} \quad \forall t \in T, s \in S^{initial} \quad (2)$$

The following two equations reflect the pre-defined trajectories during start-up and shut-down, for example during start-up procedure the plant have to go through the

estates (initial and intermediates): *RUCAPI*, *RUCAP2* and *CUCB*, staying 1 hour for each one to switch from off to on.

$$nd * W_{s,t} = \sum_{s' \in S^{down-inter}} \sum_{t'=t+1}^{t'+nd} W_{s',t'} \quad \forall t \in T, s \in S^{down-initial} \quad (3)$$

$$ns * W_{s,t} = \sum_{s' \in S^{up-inter}} \sum_{t'=t+1}^{t'+ns} W_{s',t'} \quad \forall t \in T, s \in S^{up-initial} \quad (4)$$

To model the sequence of transition between the last state of start-up and shut-down processes and the first state of on and off operation points, respectively, we present Eq. (5) and (6):

$$W_{RDCB,t} = W_{OFF1,t+1} \quad \forall t \in T \quad (5)$$

$$W_{RUCB,t} \leq W_{ON1,t+1} \quad \forall t \in T \quad (6)$$

**Critical Transition States.** In the Eq. (7)-(8) binary variable  $W_{s,t}$  is used to fulfill the critical transitions of the PSTN network. We formulate Eq. (7) to describe possible transitions that can occur from the critical states, for example the plant operates in "ONn" state at time  $t$ , then in time period  $t + 1$  it can operate in "SBI", "RDCAPI" or stay in "ONn". Note the plant can operate into a state every hour, so only a binary variable on each side of equality can be activated.

$$W_{s,t} + W_{s',t} = \sum_{s'' \in NTS} W_{s'',t+1} \quad \forall t \in T, s \in S^{critical}, s' \in LIC \quad (7)$$

Eq. (8) force to air separation process switch from intermediate state to critical state (for example when switch from "ON2" to "ONn").

$$W_{s,t} \leq W_{s',t+1} \quad \forall t \in T, s \in LIC, s' \in S^{critical} \quad (8)$$

If the plant is in the first hour of the on state ("ON1") at time  $t$ , may have been in "SBn" or "RUCB" state at time  $t - 1$ , this transition is represented below:

$$W_{SBn,t} + W_{RUCB,t} \geq W_{ON1,t+1} \quad \forall t \in T \quad (9)$$

**Production Rates and Storage Capacity Limits.** Eq. (10) and (11) capture the amount produced and amount in inventory at the end of each hour based on production rates and storage capacity limits.

$$W_{s,t} * MinP_s \leq P_{s,t} \leq W_{s,t} * MaxP_s \quad \forall t \in T, s \in S \quad (5)$$

$$Qmin \leq I_t \leq Qmax \quad \forall t \in T \quad (6)$$

**Tank Level Constraints.** The tank level, each hour and at the end of a week, are captured by Eq. (12)-(14). Note that the Eq. (12) and (13) calculate the inventory of the first hour of the horizon and the remaining hours, respectively, since the first time

period ( $t = 1$ ) has initial inventory as data. Furthermore, the plant must meet a minimum level of inventory at the end of the planning horizon (see Eq.(14)), corresponding to the last day of the week under review ( $t = 168$ ).

$$I_t = I_0 + \sum_s P_{s,t} - ED_t \quad \forall t \in T: t = 1 \quad (7)$$

$$I_t = I_{t-1} + \sum_s P_{s,t} - ED_t \quad \forall t \in T, 1 < t \quad (8)$$

$$I_t \geq MDTL_d \quad \forall t \in T, d \in D, (t, d) \in T\_D(t,d), d \in d1_d \quad (9)$$

**Power Consumption.** Amount of power consumed (fixed and variable) each hour is captured by Eq. (15).

$$PW_t = \sum_s (W_{s,t} * FPC_s + VPC_s * P_{s,t}) \quad \forall t \in T \quad (10)$$

**Objective Function.** Eq. (16) minimizes the total cost that consists of power consumption for each hour.

$$Cost = \sum_t (PW_t * EP_t) \quad (11)$$

## 4 Results

The proposed model was tested for an air separation plant with real-world electricity price and demand input data for a week (168 hours). Scenarios were defined to assess how the model faces different situations and study the impact of the optimization for different types of forecast.

The scenarios tested with the new formulation combines the input data (hourly and shift demand, and fixed, shift and hourly energy cost). In some cases the results are fixed to evaluate them with other data and to make a fair comparison (rolling-horizon model). Note that due to confidentiality, only input data corresponding to the storage capacity in the plant and daily average of demand and energy prices forecasts are shown in Table 1 and Table 2, respectively.

**Table 1.** The initial storage capacity in the plant

|                     |      |
|---------------------|------|
| Max Tank Level [kL] | 2170 |
| Min Tank Level [kL] | 850  |

**Table 2.** Daily demand and energy price forecasts

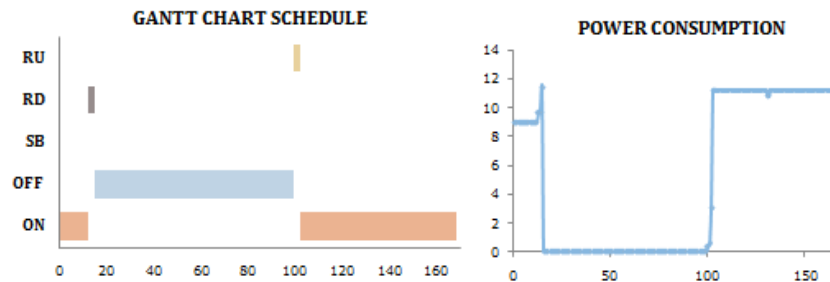
| Day | Daily demand [kL/day] | Energy Cost |
|-----|-----------------------|-------------|
| 1   | 12                    | 39.41625    |
| 2   | 15                    | 50.5875     |
| 3   | 15                    | 47.5075     |
| 4   | 14                    | 46.57375    |
| 5   | 12                    | 47.62       |
| 6   | 6                     | 47.83208    |
| 7   | 7                     | 38.76875    |

Due to confidentiality reasons, information about production levels and power consumption for the different operating modes are not disclose, only average min/max production rates are contained in Table 3. Note power consumption follows linear correlation:  $a + b * Production$ .

**Table 3.** Daily demand and energy price forecasts

|                          | Mode On | Mode Off | Mode Standby | Start-up | Shut-down |
|--------------------------|---------|----------|--------------|----------|-----------|
| Min Production [kL/hour] | 20      | 0        | 0            | 20       | 0         |
| Max Production [kL/hour] | 25      | 0        | 0            | 25       | 0         |

In the following, solutions based on flat energy cost (45.472) and time of day prices can be visualized with graphs as shown in the Figure 3 and Figure 4. When an overall average energy price is used to optimally schedule, a production plan with the minimum number of stops is obtained. Note that this plan meets inventory levels and production required by the plant while minimizing costs associated (see Figure 3).



**Fig. 3.** Solution based on flat energy cost



Furthermore, we can see in the Figure 4 that due to the hourly changing electricity prices, plant the plant operates dynamically in which transitions play an important role in the optimality of the problem. In such a way, it is economically desirable to stop producing during certain time periods to produce in others with less energy costs.

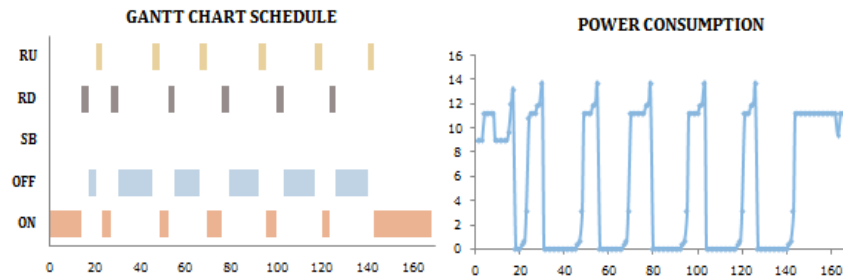


Fig. 4. Solution based on time of day prices

In Figure 5, we can compare both approaches based on on time of day prices: predictive and rolling model. In rolling-horizon scheme, due to information updates (energy price and demand) along the scheduling horizon the schedule changes in the last days of the week.

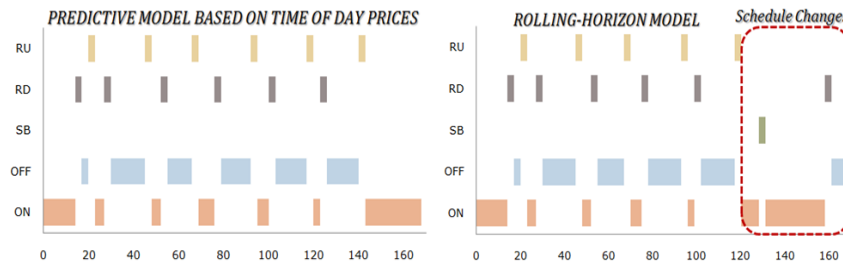


Fig. 5. Solution schedules considering both approaches (predictive and rolling-horizon)

The results in Table 4, show optimal quality solutions for the proposed methodology with a modest computational effort considering a one-hour time grid and one-week time horizon. We calculated the real total cost with the configuration of scenario 2 and real-world electricity price, to compare with result of scenario 3. The real total obtained is 41214.15. Solutions generated by using CPLEX in a PC Intel Xeon X5650 2,6 GHz.

Table 4. Results of the main scenarios

| Scenario  | Energy total cost | CPU time   |
|---|-------------------|------------|
| 1. Predictive model based on flat energy cost   | 40131.82          | 5.454 sec. |
| 2. Predictive model based on time of day prices | 35524.16          | 0.093 sec. |
| 3. Rolling-horizon model                        | 39240.04          | 0.109 sec  |

## 5 Conclusions

Based on the preliminary results achieved, it can be concluded that the predictive MILP-based scheduling approach looks very efficient and robust. The model is able to consider all problem features and easy to adapt to reactive scheduling (a rolling horizon approach). Therefore, the developed model is promising for solution scheduling for the application to real-world air separation industrial plants.

The PSTN model is easy adapted to other plant configurations, including the identified additional features. It allows evaluation of daily and hourly reactive decisions based on energy price changes (day-ahead market and imbalance market).

## References

1. Mitra S., Sun L., Grossmann I.E.: Optimal scheduling of industrial combined heat and power plant under time-sensitive electricity prices. *Energy* 54, 194–211 (2013)
2. Karwan M. H., Kebli M. F.: Operations planning with real time pricing of a primary input. *Comput. & Operations Research* 34, 848-867 (2007)
3. Mitra S., Grossmann I.E., Pinto J.M., Arora N.: Optimal production planning under time-sensitive electricity prices for continuous power-intensive processes. *Comput. Aided Chem. Eng.* 38, 171–84 (2012)
4. Hadera H., Harjunoski I., Grossmann I.E., Sand G, Engell S.: Steel production scheduling under time-sensitive electricity cost. *Comput. Aided Chem. Eng.* 33, 373–8 (2014)
5. Castro, P. M., Harjunoski, I., Grossman, I. E.: Rolling-Horizon Algorithm for Scheduling under Time-Dependent Utility Pricing and Availability. *Comput. Aided Chem. Eng.* 28, 1171–1176 (2010)
6. Nolde K., Morari M.: Electrical load tracking scheduling of a steel plant. *Comput. Aided Chem. Eng.* 34(11), 1899–903 (2010)